Identification of Time Delays in Flight Measurements

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A computer program has been developed for the maximum likelihood estimation of parameters in general nonlinear systems. Sensitivity matrix elements are calculated numerically, overcoming the need for explicit sensitivity equations. Parameters such as break points and time delays are successfully determined using simulated data. Two examples using aircraft flight data are shown to demonstrate the identification of multiple time delays concurrently with other parameters.

Introduction

IT is well known that time delays or phase shifts are often present in flight measurements due to a number of possible causes. For example, data sampling techniques can lead to time delays between sequentially sampled variables. Time delays can also be introduced by preprocessing of data prior to recording. Phase shifts can be introduced by instrumentation filters in the signal conditioning electronics or by lags in the response of instrumentation, especially air data systems. Provided the measurement system phase response is linear over the range of interest, phase shifts can be approximated by simple time delays. Equivalent time delays of several tenths of a second are not uncommon, due to one or a combination of the aforementioned causes.

Small time delays in flight data can have a large effect on parameters extracted using techniques such as maximum likelihood estimation. For example, a time delay of 0.1 s can typically result in a 50% error in stability and control derivatives.[1] In data compatibility checking applications, large variations in estimates of instrument bias errors have been obtained, due to relative time delays in data records.[2]

In another type of application, the approximation of high-order dynamic systems by a lower order equivalent model can be facilitated by using a time delay to account for the higher order effects. Application to helicopter dynamics has led to estimated time delays of several tenths of a second.[3]

If time delays are known, it is possible to adjust the data accordingly, prior to further analysis. However, the magnitude or even the presence of delays may not be apparent. It is desirable in these cases to be able to identify possible time delays as part of the analysis. In the frequency domain, estimation of time delays is relatively straightforward[4,5] but is limited in practice to linear systems.

In the time domain, the maximum likelihood parameter estimation technique is widely used to determine aircraft flight parameters from flight test data.[6] Although much of the effort in this field has been confined to the analysis of linear systems, the method is equally applicable to nonlinear problems. For example, a maximum likelihood computer program developed by one of the authors has been successfully used to determine accelerometer offsets and calibration errors using the nonlinear equations for aircraft kinematics.[7] The sensitivity matrix elements required for the solution were calculated explicitly by mathematical differentiation of the state equations, often a long and tedious process. This also results in a sizeable proportion of the program being problem-specific. At the same time, sensitivities for some types of nonlinearities, such as break points or time delays, cannot be obtained explicitly.

In this paper, a maximum likelihood parameter estimation computer program for general nonlinear systems is outlined. Sensitivities are calculated numerically by finite differences, overcoming the need for explicit sensitivity equations. Similar programs for generalized nonlinear systems have been described elsewhere.[8,9] However, their specific application to the estimation of break points or time delays does not appear to have been reported. The program described here has been used successfully in both situations. In particular, the importance of determining time delays is demonstrated here in applications using both fixed and rotary wing flight data.

In the next section, a brief outline of the maximum likelihood method for nonlinear systems is provided, including the procedure used to obtain sensitivities numerically. Using simulated time histories of landing gear load and deflection, the use of the program to estimate both break points and time delays simultaneously in a nonlinear system is demonstrated. Finally, two applications involving the identification of time delays in flight data are presented to illustrate the importance of their effects and the means for accounting for them in the normal data analysis process.

Description of Method

Assume that the system can be described in general by a set of nonlinear dynamic equations of the form:

$$\dot{x}(t) = f(x(t), u(t), \xi) \tag{1}$$

$$y(t) = g(x(t), u(t), \xi) \tag{2}$$

$$z(t_i) = y(t_i) + n(t_i) \tag{3}$$

where

x = state vector
u = control input vector
y = observation vector
z = measurement vector, sampled at N discrete time points t_i for $i = 1, \ldots, N$
n = measurement noise vector, assumed to be white Gaussian with zero mean
ξ = vector of unknown parameters

The maximum likelihood method determines the most probable value of ξ by maximizing a likelihood function that is defined as the probability of obtaining output z given parameter ξ. Full details of the method can be found in standard references[10] and amount to minimization of a cost

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functional $J(\xi)$ given by

$$J(\xi) = \frac{1}{2}\sum_{i=1}^{N} [z(t_i) - y(t_i)]^T R^{-1}[z(t_i) - y(t_i)] \tag{4}$$

with an estimate of R, the covariance of residuals, given by

$$R = \frac{1}{N}\sum_{i=1}^{N} [z(t_i) - y(t_i)][z(t_i) - y(t_i)]^T \tag{5}$$

A modified Newton-Raphson algorithm is used to achieve the minimization iteratively. Starting with an initial estimate, ξ_n, for ξ, revised estimates are obtained from

$$\xi_{n+1} = \xi_n - \Delta\xi \tag{6}$$

where $\Delta\xi$, the change in ξ per iteration, is given by

$$\Delta\xi = [\nabla_\xi^2 J]^{-1}[\nabla_\xi J] \tag{7}$$

with

$$\nabla_\xi^2 J = \sum_{i=1}^{N} (\nabla_\xi y(t_i))^T R^{-1}(\nabla_\xi y(t_i)) \tag{8}$$

$$\nabla_\xi J = \sum_{i=1}^{N} (\nabla_\xi y(t_i))^T R^{-1}(z(t_i) - y(t_i)) \tag{9}$$

Starting with a priori values for elements of the parameter vector ξ, initial values of $y(t_i)$ are obtained in Eq. (2), leading to an initial value of R [Eq. (5)]. From Eq. (7) we deduce $\Delta\xi$ and, hence, an improved value of ξ, which is used to obtain a new $y(t_i)$ and, thus, improved R. This process is repeated until the required convergence is achieved. Computation of $\Delta\xi$ requires at each time point t_i 1) values of the measurement vector $z(t_i)$; 2) values of the observation vector $y(t_i)$; and 3) the sensitivity matrix $\nabla_\xi y(t_i)$.

Values of the measurement vector are read in as data. Computation of the current observation vector $y(t_i)$, from Eq. (2), requires current state vector values, which are obtained by numerical integration of the assumed system state equations [Eq. (1)]. A fourth-order Runge-Kutta numerical integration procedure is adopted.

An indication of the error in each parameter is given by the Cramer-Rao lower bound,[10] which is the covariance σ, given by

$$\sigma(\xi) = \left[\sum_{i=1}^{N} (\nabla_\xi y(t_i))^T R^{-1}(\nabla_\xi y(t_i))\right]^{-1} \tag{10}$$

The sensitivity matrix elements are here approximated by numerical differences. This approach overcomes the need for explicit sensitivity equations, which are not always easy to determine. The forward difference method has been shown to give adequate results,[8] and is adopted here, requiring evaluation of state and observation variables at two parameter values, $\xi + \delta\xi$ and ξ. The jkth element of the sensitivity matrix, $\nabla_\xi y(t_i)_{jk}$, is given by

$$\nabla_\xi y(t_i)_{jk} = \frac{\partial y_j}{\partial \xi_k} \approx \frac{y_j(\xi_k + \delta\xi_k) - y_j(\xi_k)}{\delta\xi_k} \tag{11}$$

where y_j and ξ_k are components of vectors y and ξ.

The preceding procedure takes account of most parameter types, including break points (see below). However, the case of unknown time delays as parameters is somewhat different and a separate procedure should be followed. Given that the kth parameter ξ_k is a time delay τ, in observation y_j, then Eqs. (1–3) are first calculated at time $\hat{t}$, the time *without* any time delays. Then, y_j requires reevaluation at translated time t to obtain y_j^*:

$$y_j^*(t) = y(\hat{t} - \tau) \tag{12}$$

The sensitivity matrix elements for parameter τ are calculated as

$$\frac{\partial y_j^*}{\partial \tau} \approx \frac{y_j^*(\tau + \Delta t) - y_j^*(\tau)}{\Delta t} \tag{13}$$

where the incremental parameter value used in the forward difference calculation $\delta\tau$ is taken as Δt, the time interval between successive measurements $z(t_i)$. Using Eqs. (5) and (7), $\Delta\tau$ and, hence, a new estimate for parameter τ, is determined. This procedure is repeated until the required convergence is achieved. The time delay parameters were unconstrained during each iteration; however, prior to each new iteration, they were rounded to the nearest multiple of Δt. This step was essential so that observations y, calculated at time $\hat{t} - \tau$, are able to be compared directly with measurements z, which are known only at time t_i, a multiple of Δt.

An alternative approach, which was examined in some detail, is to include as an a priori constraint the restriction that the time delay is a multiple of Δt. This has the effect of adding a term $[\boldsymbol{\tau} - \boldsymbol{n}\,\Delta t]^T D[\boldsymbol{\tau} - \boldsymbol{n}\,\Delta t]$ to the cost functional J [Eq. (4)], where D is a weighting matrix, $\boldsymbol{\tau}$ is a vector of time delays τ_ξ, and $\boldsymbol{n}$ is a vector of integers n_ξ, such that $n_\xi\,\Delta t$ is the closest multiple of Δt to τ_ξ. This procedure is outlined in Ref. 11 for general a priori constraints, and in this case results in alterations to Eqs. (8) and (9), as follows:

$$\nabla_\xi^2 J = \sum_{i=1}^{N} (\nabla_\xi y(t_i))^T R^{-1}(\nabla_\xi y(t_i)) + D \tag{14}$$

$$\nabla_\xi J = \sum_{i=1}^{N} (\nabla_\xi y(t_i))^T R^{-1}(z(t_i) - y(t_i)) + D(\boldsymbol{\tau} - \boldsymbol{n}\,\Delta t) \tag{15}$$

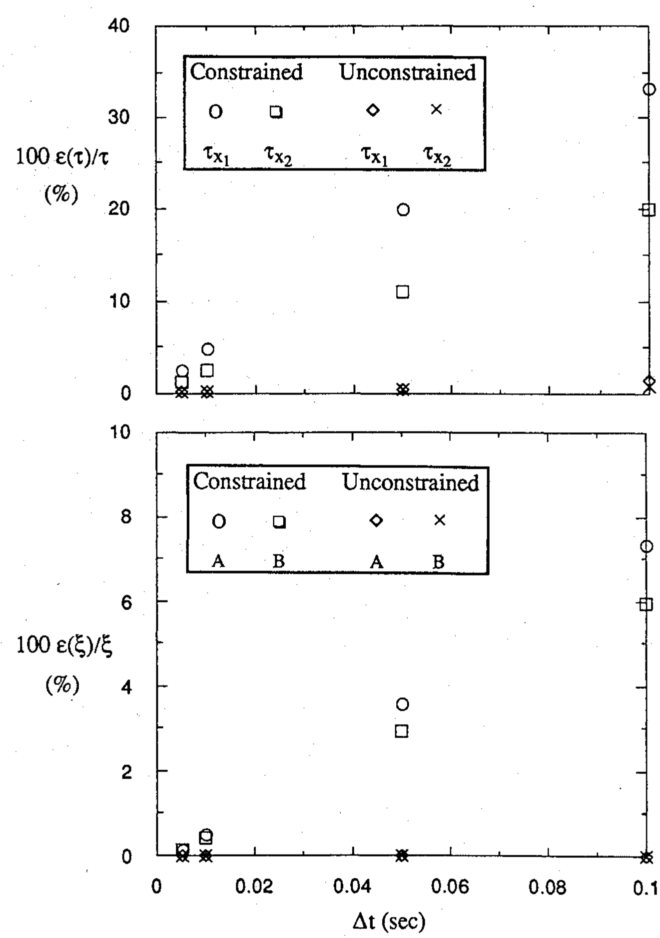

Fig. 1 Comparison of unconstrained and a priori constrained time delay methods for time delays midway between measurement time points.

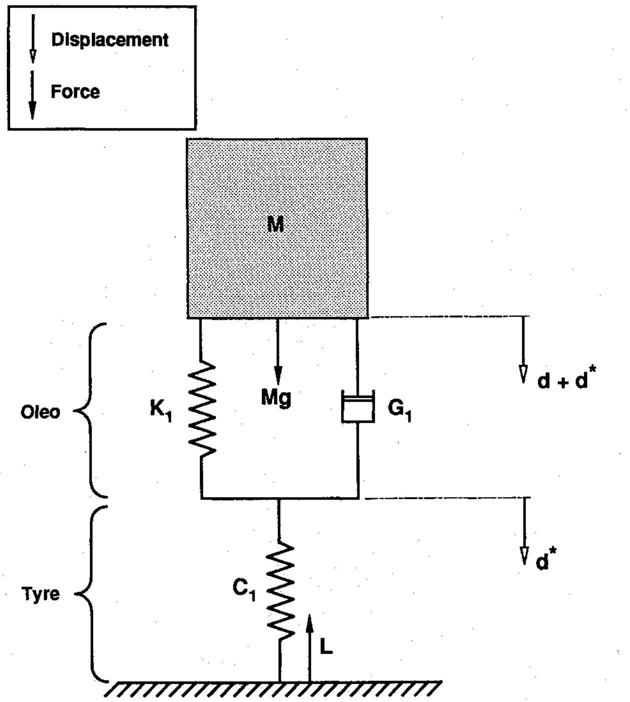

Fig. 2 Forces and displacements on landing gear.

with consequent change in $\Delta\xi$ [Eq. (7)]. Note that D is a diagonal matrix of order $N_\xi \times N_\xi$, where N_ξ is the number of parameters being estimated. The diagonal elements of D are zero, if the parameter represented is not a time delay, and take value k, if the parameter is a time delay. Large k ensures that the a priori constraint dominates the other terms in Eqs. (14) and (15). A large value of $k = 99999$ was used to ensure that the time delays are a multiple of Δt. The a priori constraint method was tried out using simulated data for a simple model (see below) and led to slightly reduced Cramer-Rao lower bounds than the unconstrained approach. However, when time delays were not multiples of Δt, not only were estimates of the time delays less accurate, but also estimates of other parameters were less well determined, since constrained time delays were used in determining their values. This effect was particularly noticeable if Δt was large (~ 0.1). Time delays as large as 0.1 are not uncommon in flight data recording. To illustrate the abovementioned comparison, the following simple state model was used:

$$\dot{x}_1 = A(x_1 - 2.5) \tag{16}$$

$$\dot{x}_2 = Bx_2 \tag{17}$$

with 5 s of simulated data created using $A = -1.5$, $B = -1.3$, and variable time steps Δt ranging from 0.005 to 0.1. Initial conditions were $x_1 = 1.0$ and $x_2 = 2.0$. Time delays that were multiples of Δt ($\tau_{x1} = 0.1$ and $\tau_{x2} = 0.2$) and also midway between time points ($\tau_{x_1} = 0.1 + \Delta t/2$ and $\tau_{x_2} = 0.2 + \Delta t/2$) were simulated and resulting parameter estimates were determined over five iterations, using both the unconstrained approach and the a priori constraint approach. A priori values for the parameters were -1.0 for A and B, and 0.0 for τ_{x_1} and τ_{x_2}. For the case of time delays midway between measurement time points, the error in each parameter estimate $\varepsilon(\xi)$, as a percentage of the exact parameter ξ, is plotted against the time step used, Δt, for time delay parameters and other parameters (Fig. 1). The superior results given by the unconstrained method are clearly demonstrated. For the case of time delays that are exact multiples of measurement time points, the two methods led to similar results, with errors $\varepsilon(\xi)/\xi$ less than 0.03% for parameters A and B. For $\varepsilon(\tau)/\tau$, the error was zero by definition for the constrained approach, whereas the unconstrained method led to small errors $\varepsilon(\tau)/\tau < 0.2\%$, slightly less than those shown in Fig. 1. Thus, the unconstrained method is preferable to the constrained approach, except in the unlikely event that the time delay is very close to a multiple of Δt.

A possible way to avoid the problem of constraining time delays to be multiples of Δt is to first obtain an estimate for time delay τ_ξ, then use interpolation to determine the measurement data z at time $t_i - \tau_\xi (i = 1,\ldots,N)$. This way, z and y can still be compared directly without restriction on τ_ξ. This procedure would involve substantial extra computation for cases of several different time delays, and due to the excellent results obtained for the simpler unconstrained time delay method, it was decided not to adopt the interpolation approach and to use the unconstrained time delay method.

Example Using Simulated Data

An application involving break points and time delays, both highly nonlinear parameters, is discussed.

Consider the case of an aircraft landing gear drop test. The landing gear comprises a large mass M attached to a wheel and tire by an oleo that acts as a combined spring and shock absorber. The oleo is modeled as a massless, nonlinear damped spring of a two-stage type. For oleo deflection d, greater than some value d_o, a second, stiffer, more damped "spring" is activated. In this way, extra hard landings are catered for, whilst soft landings do not suffer from overdamping. Essentially a different set of state equations exist for $d > d_o$, with d_o termed as a *break point*.

The load on the oleo L_o is given by

$$L = C_1 d^* = \begin{cases} K_1 d^2 + G_1 \dot{d} & (\text{for } d < d_o) \\ K_1 d_o^2 + K_2 (d - d_o)^2 + G_2 \dot{d} & (\text{for } d \geq d_o) \end{cases} \tag{18}$$

The tire is modeled as a massless, linear undamped spring with load L_t given by

$$L_t = C_1 d^* \tag{19}$$

where d^* is the tire compression. Since the tire and oleo are assumed to be massless, the oleo load is equal to the tire load; thus, $L_o = L_t \equiv L$ (see Fig. 2).

The equation of motion for the system is

$$M(\ddot{d} + \ddot{d}^*) = Mg - L \tag{20}$$

Taking d, d^*, and $w\,(=\dot{d} + \dot{d}^*)$ as state variables, the following state equations are obtained:

$$\dot{w} = g - \frac{C_1 d^*}{M} \tag{21}$$

$$\dot{d} = \frac{C_1 d^* - K_1 d_1^2 - K_2 d_2^2}{G} \tag{22}$$

$$\dot{d}^* = w - \dot{d} \tag{23}$$

where

$$\left.\begin{aligned} d_1 &= d \\ d_2 &= 0 \\ G &= G_1 \end{aligned}\right\} d < d_0 \qquad \left.\begin{aligned} d_1 &= d_o \\ d_2 &= d - d_o \\ G &= G_2 \end{aligned}\right\} d \geq d_o$$

and where d_1 is the first-stage deflection and d_2 the second-stage deflection. Observation variables are the load and oleo deflections, given by

$$L_{\text{obs}}(\hat{t}) = C_1 d^*(\hat{t}) \tag{24}$$

$$d_{\text{obs}}(\hat{t}) = d(\hat{t}) \tag{25}$$

where $\hat{t}$ is the time without any time delays.

Table 1 Parameter estimates for 8-parameter model (including break point and time delays)

Parameter	A priori value	True value	Maximum likelihood parameter value
K_1, $Nm^{-2}(\times 10^5)$	2	4	4.03 ± 0.03
K_2, $Nm^{-2}(\times 10^5)$	20	45	47.54 ± 2.0
G_1, $Nsm^{-1}(\times 10^4)$	1.5	2.5	2.49 ± 0.02
G_2, $Nsm^{-1}(\times 10^4)$	3	4	3.93 ± 0.05
C_1, $Nm^{-1}(\times 10^5)$	5	7	7.04 ± 0.11
d_o, m	0.1	0.23	0.229 ± 0.002
τ_d, s	0	0.07	0.070 ± 0.0008
τ_L, s	0	0.09	0.090 ± 0.0005

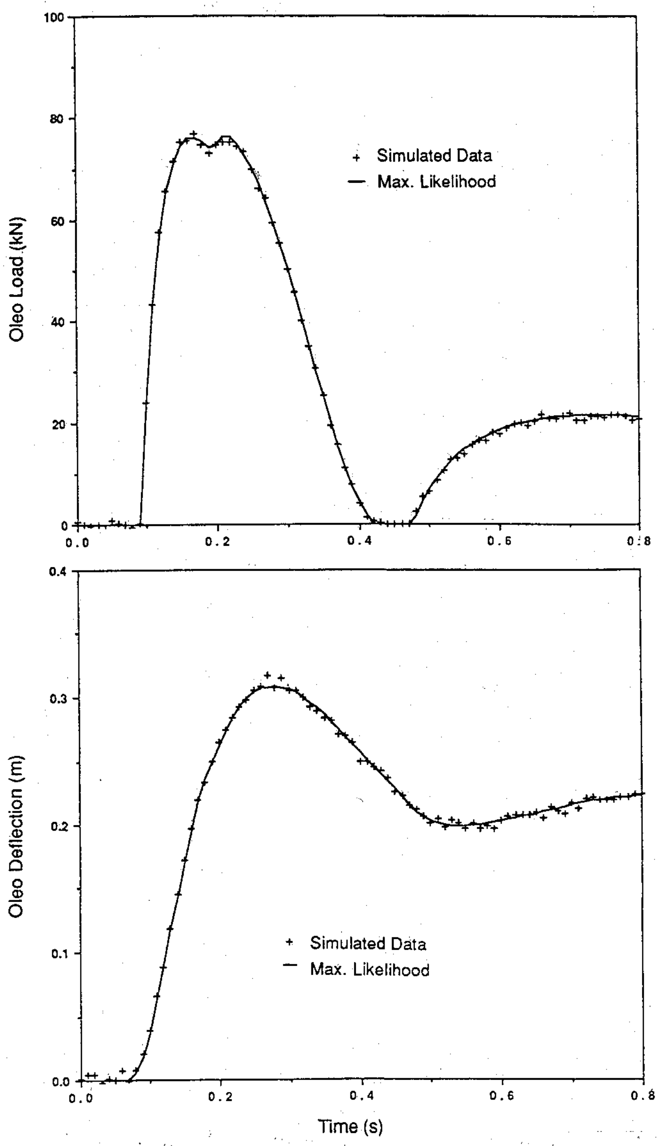

Fig. 3 Oleo load and displacement for simulated aircraft landing gear drop test: $w_o = 4\ m\ s^{-1}$, $M = 2000$ kg.

There are no control inputs u in this example. The unknown parameters are K_1, K_2, G_1, G_2, C_1, and break point d_o. Equations (21–25) are subject to constraints that d, d^*, and L are all ≥ 0. If unknown time delays are present in the data, they can be included in the maximum likelihood procedure as additional parameters. Time delays in d or L result in translations of the preceding observations:

$$d_{obs}(t) = d(\hat{t} - \tau_d) \tag{26}$$

$$L_{obs}(t) = L(\hat{t} - \tau_L) \tag{27}$$

where τ_d and τ_L are the time delays in d and L, respectively.

The landing gear is dropped with initial velocity w_o, so initial values of w, d, and d^* are w_o, 0, and 0 respectively.

Using simulated time histories of oleo load and deflection, with 81 data points and a time interval of $\Delta t = 0.01$ s, the maximum likelihood method is applied with numerical sensitivity matrix computations. Zero mean Gaussian noise with an rms of 0.0025 m for d and 0.5 kN for L is superimposed on the simulated data.

After 10 iterations, excellent agreement with the simulated measurements is obtained (Fig. 3). The parameter values are given in Table 1, with the *true values* being the values used in the simulation, and the *a priori values* being the initial guess of these values. All parameters are well identified, including the break point and time delays. The Cramer-Rao error bounds are also shown in this and all subsequent tables.

It should be noted that careful consideration needs to be given to the selection of the a priori value of the break point parameter d_o. Clearly, if $d_o > d_{max}$ (where d_{max} is the maximum value reached by quantity d in the time interval under consideration), either initially or during one of the early iterations (when parameter values are liable to oscillate), then any small change in d_o will have no effect on the observation vector y. Consequently, sensitivities $\partial y_j / \partial d_o$ will be zero, resulting in no improved estimated of d_o. A suitable initial value for d_o can usually be obtained by experimentation. A priori values of time delays are usually set to zero, since time delays are expected to be small.

For most parameters, the size of $\delta\xi_k$ used in the numerical sensitivity calculations should be significantly less than the size of ξ_k. We use here $\delta\xi_k/\xi_k = 10^{-3}$. However, break points or time delays are special cases, and careful consideration needs to be given to the size of $\delta\xi_k$. This is because measurements z only exist at discrete time points t_i, and observations y are only calculated at these time points. For break point d_o, in order that $\partial y_j / \partial d_o$ is nonzero for at least one t_i, any change δd_o in the value of d_o must be large enough such that the time where the break point acts, $t(d_o)$ moves across at least one point. If $t(d_o)$ is within the time interval $[t_i, t_{i+1}]$, then for δd_0 to result in nonzero sensitivities, we require

$$|t(d_o) - t(d_o - \delta d_o)| > t(d_o) - t_i \tag{28a}$$

or

$$|t(d_o + \delta d_o) - t(d_o)| > t_{i+1} - t(d_o) \tag{28b}$$

(see Fig. 4).

The size of δd_o necessary to satisfy the preceding condition can be found by experimentation. In the case reported here, $\delta d_o = 0.01$ was sufficient.

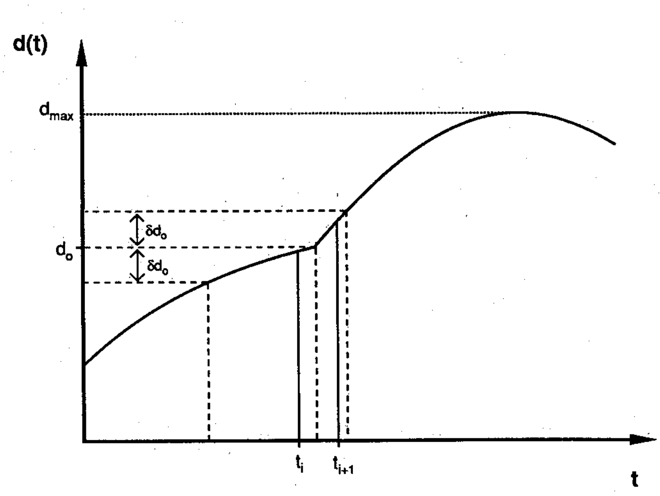

Fig. 4 Effect of shifting a break point d_o by δd_o.

Table 2 Parameter estimates using fixed-wing flight test data

Parameter	A priori value	Maximum likelihood value without time delays	Maximum likelihood value with time delays
C_{M_α}, deg^{-1}	-0.050	-0.036 ± 0.0002	-0.041 ± 0.00008
C_{M_q}, rad^{-1}	-30	-9.92 ± 0.46	-8.29 ± 0.14
C_{M_δ}, deg^{-1}	-0.035	-0.019 ± 0.0002	-0.022 ± 0.00009
C_{M_o}	0	0.102 ± 0.0007	0.116 ± 0.0003
C_{N_α}, deg^{-1}	0.09	0.071 ± 0.0009	0.071 ± 0.0003
C_{N_o}	0	-0.093 ± 0.003	-0.092 ± 0.0008
$C_{N_o} + \hat{\alpha}_o$	-0.25	-0.087 ± 0.004	-0.093 ± 0.001
τ_α, s	0	——	0.030 ± 0.001
τ_q, s	0	——	0.058 ± 0.0007
τ_{a_n}, s	0	——	0.075 ± 0.0007

Results Using Measured Test Data

Measured test data were available for both fixed and rotary winged aircraft undergoing dynamic maneuvers. The maximum likelihood program developed here was applied to two examples chosen to illustrate a range of applications. Results are outlined below.

Fixed-Wing Aircraft Longitudinal Maneuver

Fixed-wing flight test data were available for an aircraft undergoing a longitudinal maneuver. State variables are taken as angle of attack α, pitch angle θ, and pitch rate q, while the control input is the elevator deflection δ. Observation variables, for which measurements are given, are α, q, and normal acceleration a_n.

The full state equations for a longitudinal maneuver are given in Eq. (55) of Ref. 12. For small α and ϕ, we obtain the state equations:

$$\dot{\alpha} = \frac{-\bar{q}SR}{mV}(C_N + \hat{\alpha}_o) + q + \frac{gR}{V}\cos\theta \tag{29}$$

$$\dot{q} = \frac{\bar{q}ScR}{I_y} C_M \tag{30}$$

$$\dot{\theta} = q \tag{31}$$

with the force coefficient

$$C_N = C_{N_\alpha}\alpha + C_{N_q}\frac{qc}{2VR} + C_{N_\delta}\delta + C_{N_o}$$

and moment coefficient

$$C_M = C_{M_\alpha}\alpha + C_{M_q}\frac{qc}{2VR} + C_{M_\delta}\delta + C_{M_o} + C_{M_{\dot{\alpha}}}\frac{\dot{\alpha}c}{2VR}$$

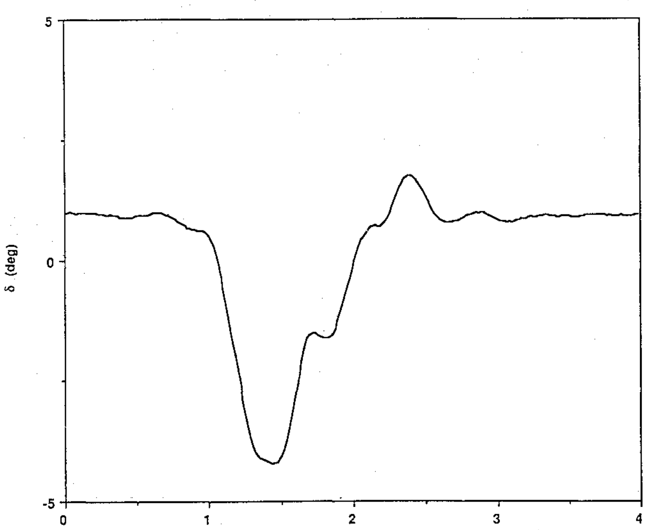

Fig. 5 Control input (elevator deflection)—fixed-wing flight test.

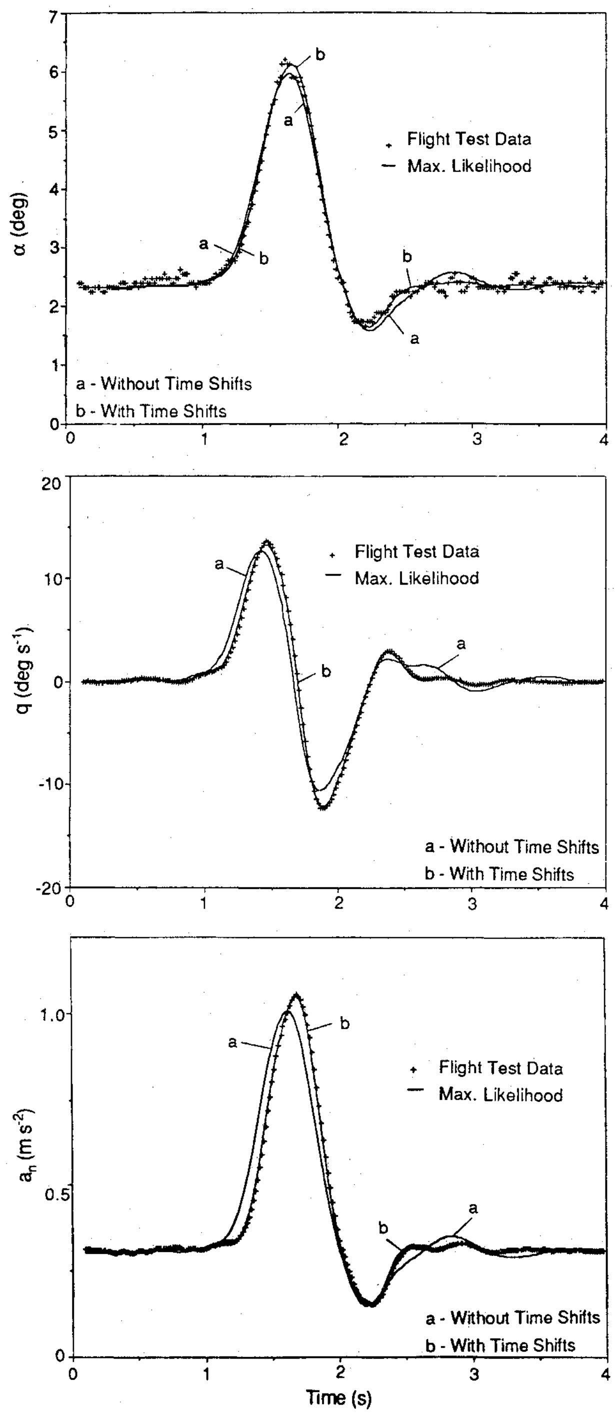

Fig. 6 Angle of attack, pitch rate, and normal acceleration—fixed wing flight test.

where c, m, S, and V are the reference chord, mass, wing area, and velocity of the aircraft, respectively, $\hat{\alpha}_o$ is a dimensionless bias, and $\bar{q}$ is the dynamic pressure.

Observations [Eq. (56) of Ref. 12] are

$$\alpha_{\rm obs} = \alpha - \frac{X_\alpha}{V} q \tag{32}$$

$$q_{\rm obs} = q \tag{33}$$

$$a_{n_{\rm obs}} = \frac{\bar{q}S}{mg} C_N + \frac{X_{a_n}}{gR}\dot{q} + \frac{Z_{a_n}}{R^2 g} q^2 \tag{34}$$

R is simply the radian-to-degree conversion factor, X_{a_n} and X_α are the longitudinal instrument offsets from the aircraft c.g., and Z_{a_n} is the vertical instrument offset from the aircraft c.g.

Unknown parameters are C_{M_α}, C_{M_q}, C_{M_δ}, C_{M_e}, C_{N_α}, C_{N_o}, and $C_{N_o} + \hat{\alpha}_o$. Parameter $C_{M_{\dot{\alpha}}}$ is also unknown, but in the maneuver reported here it is difficult to determine both $C_{M_{\dot{\alpha}}}$ and C_{M_q} independently. Consequently, the two parameters are linked by a factor, obtained from their a priori values:

$$C_{M_{\dot{\alpha}}} = 0.2836 \times C_{M_q} \tag{35}$$

The maximum likelihood procedure is applied *without* time delays, with a time interval of $\Delta t = 1/60$ s and 10 iterations, and with the control input variation shown in Fig. 5. Results are shown in Fig. 6 and Table 2. It is noted that there are discrepancies between actual measurements and predicted observations, which appear to be due to time lags. Application of the maximum likelihood procedure with three additional time delay parameters, τ_α, τ_q, and τ_{a_n}, leads to marked improvements in the predicted observation variables (Fig. 6).

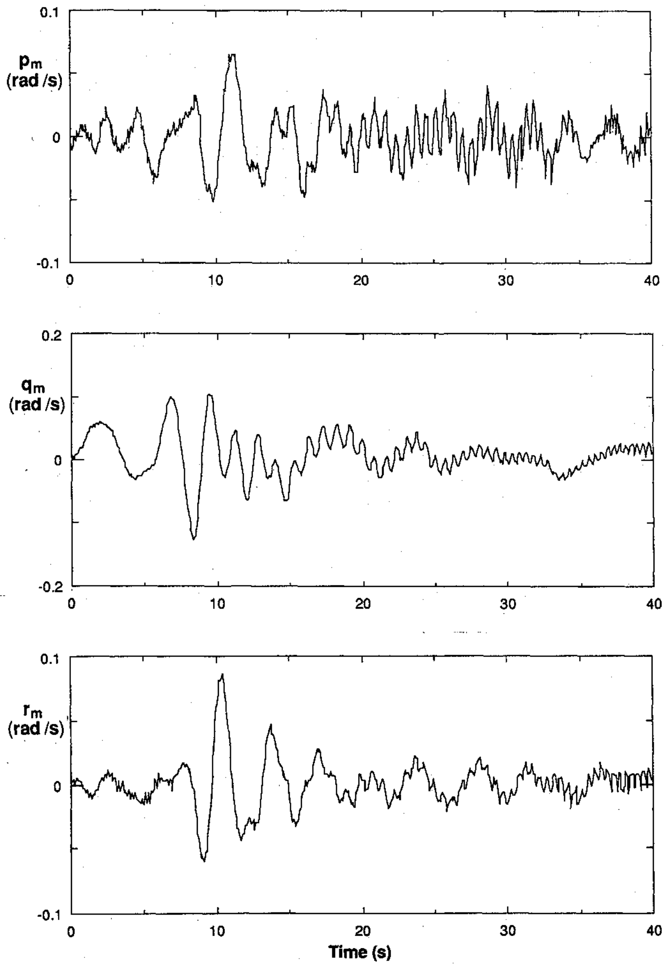

Fig. 7 Roll, pitch, and yaw rates—data compatibility check.

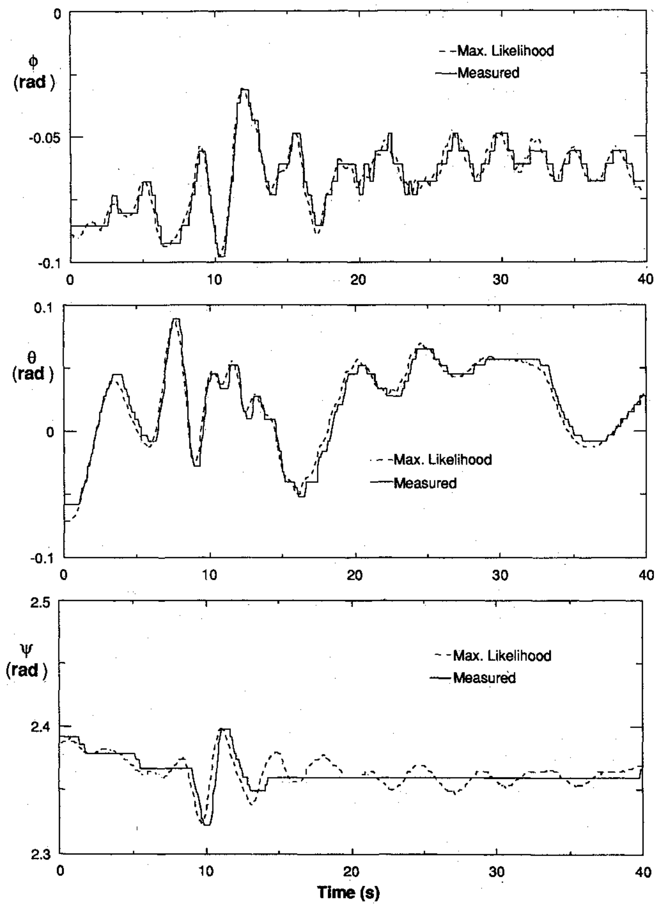

Fig. 8 Roll, pitch, and yaw response (no time shifts included)—data compatibility check.

The Cramer-Rao error bounds are also reduced (Table 2), indicating improved parameter identification.

Flight Data Compatibility Checking

In this application, redundancies in the flight measurements are used to correct systematic bias and scale factor errors in the instrumentation and also to reconstruct faulty or missing data records.[8] In general, the six-degree-of-freedom kinematic equations are applicable, but when only the attitudes and rates are of interest, the three equations relating these quantities can be conveniently decoupled from the velocity equations. The relevant equations are[13]

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \tag{36}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{37}$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta \tag{38}$$

In this set of nonlinear equations, the measured roll, pitch, and yaw rates (p, q, and r) are treated as inputs; the respective attitudes (ϕ, θ, and ψ) are the computed outputs, which are required to match with their observed values. Sources of error include both bias and scale factor errors in the measured inputs and in the observed outputs. The error model for the inputs has the form

$$p = (1.0 + \lambda_p)p_m + b_p \tag{39}$$

where b_p and λ_p are the bias and scale factor errors, respectively, and subscript m refers to the measured value. Similar equations apply for q and r. Note that Eq. (39) does not allow for random errors. This is usually a reasonable assumption, given the accuracy of the rate gyro instrumentation. The error

Table 3 Parameter estimates from data compatibility check

Parameter	Maximum likelihood value without time delays	Maximum likelihood value with time delays
b_p, rad s^{-1}	0.0012 ± 0.000013	0.0013 ± 0.000008
b_q, rad s^{-1}	-0.0051 ± 0.000021	-0.0051 ± 0.000017
b_r, rad s^{-1}	-0.00054 ± 0.000052	-0.00050 ± 0.000021
b_ϕ, rad	-0.030 ± 0.0032	-0.034 ± 0.020
b_θ, rad	0.152 ± 0.0063	-0.017 ± 0.0051
τ_p, s	—	0.182 ± 0.005
τ_q, s	—	0.120 ± 0.006
τ_r, s	—	0.346 ± 0.012
rms fit error		
$\phi_{\rm rms}$, rad	0.00365	0.00254
$\theta_{\rm rms}$, rad	0.00685	0.00535
$\Psi_{\rm rms}$, rad	0.00797	0.00594

model for the observations is of the form

$$\phi_m = (1.0 + \lambda_\phi)\phi + b_\phi + n_\phi + (\text{time delay})_\phi \qquad (40)$$

where a random noise term n_ϕ is now included and allowance is also made for a possible time delay. Similar equations apply for θ and ψ.

Thus, the unknown parameters to be identified may include up to six bias errors, six scale factor errors, and three time delay terms.

The flight data used are for a helicopter dynamic maneuver in response to a longitudinal cyclic frequency sweep at about 130 knots. (Data provided by courtesy of McDonnell Douglas Helicopter Company.) Typically, all degrees of freedom are excited, so that decoupling of longitudinal and lateral motions is not possible. The p, q, and r time histories are shown in Fig. 7 and are seen to be well defined, with reasonable signal to noise ratio. The matched results, excluding time delays, are shown in Fig. 8, and with time delays identified, in Fig. 9, which indicates a considerable improvement in match. In both cases, bias errors have been allowed for all measurements, but scale factor errors have been assumed to be zero. In addition, initial conditions for ϕ, θ, and ψ were identified as additional unknowns because of the difficulty of inferring them from the measurements. The relatively poor resolution of the ϕ, θ, and ψ measurements, especially yaw attitude ψ, makes data reconstruction an attractive option in this case.

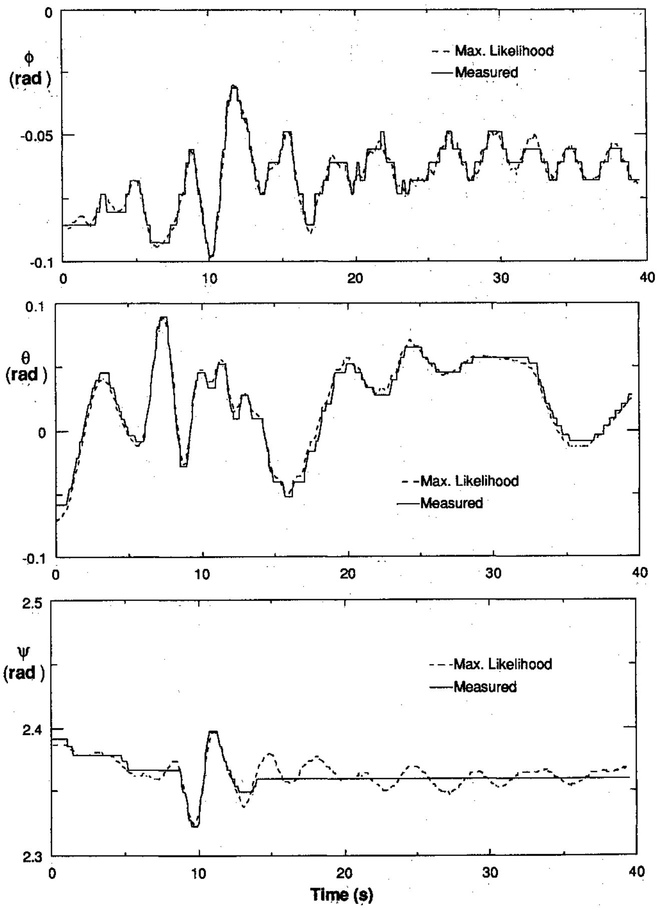

Fig. 9 Roll, pitch, and yaw response (time shifts included)—data compatibility check.

Table 3 summarizes the identification results obtained from 40 s of record sampled at 20 samples/s, after 12 iterations of the maximum likelihood program. An a priori value of zero was assumed for all biases. The results show that significant time delays are apparent in all of the outputs. At the same time, a large change in the identified value of b_θ is obtained, from an unrealistic value of 0.152 rad to a more reasonable -0.017 rad. The identified time delay values are also useful information in relation to handling qualities considerations.

Concluding Remarks

A general maximum likelihood program, suitable for identification of parameters in nonlinear systems, has been described. By calculating sensitivities numerically, identification of parameters such as break points or time delays is made possible, and has been illustrated using simulated data.

The ability to identify multiple time delays concurrently with other parameters in a nonlinear system is particularly useful in the analysis of aircraft flight data. Two examples using both fixed and rotary wing data have been used to illustrate the possible range of application. The results confirm the important effect time delays can have on other identified parameters, and demonstrate the feasibility of identifying multiple time delays as an integral part of flight data analysis procedures, for both linear and nonlinear systems.

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